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Three-dimensional integral dry friction model for the motion of a rectangular body

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Introduction. A three-dimensional dry friction model in the interaction of a rectangular body and a horizontal rough surface is considered. It is assumed that there is no separation of the body from the horizontal surface. The body motion occurs under the conditions of combined dynamics when, in addition to the longitudinal movement, the body participates in twisting.
Materials and Methods. Linear fractional Pade approximations are proposed, which replaced the cumbersome analytical expressions that most accurately describe the motion of bodies on rough surfaces. New mathematical models describing sliding and twisting of bodies with a rectangular base are proposed.
Results. Analytical expressions of the principal vector and moment of friction for rectangular contact areas are developed and scientifically established. A friction model that takes into account the relationship between sliding and twisting speeds, which provides finding solutions for Pade dependences, is developed. After numerical solution to the equations of motion, the dependences of the sliding speed and angular velocity on time were obtained and constructed. Graphs of the dependences of the friction forces and their moment on two parameters (angular velocity and slip velocity) were constructed, which enabled to compare the integral and normalized models of friction. The comparison results showed good agreement of the integral model and the model based on Pade approximations.
Discussion and Conclusions. The results obtained provide considering the dynamic coupling of components, which determines the force interaction of a rectangular body and a horizontal surface. These results can be used in mobile robotics. The analyzed motion of the body occurs through the motion control of a material point inside the body. Such mobile robots can be used when solving a wide class of problems: when creating autonomous robots for the exploration of outer space and planets; in the diagnosis and treatment in case of passing through complex structures of veins and arteries; in research under water, in places of large differential temperature; in underground operations.

Keywords: dry friction, rectangular body, solid body, dynamics, sliding, twisting, friction force, Pade approximations.

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Introduction. The study of the movement of a rectangular body is a challenge in the mobile robotics [1]. This movement is due to the control of the material point inside the body. Such mobile robots can be used to solve a wide range of tasks. For example, when creating autonomous robots for the exploration of outer space and planets; for medical purposes, in diagnosis and treatment, for example, in case of passing through complex structures of veins and arteries; as well as for underground work and research under difficult conditions, for example, under water and in places of large differential temperature [1, 2].

Thus, more and more challenges are being set for robotics, which require theoretical research, including studying models of friction between the body and the surface under the conditions of combined dynamics [3, 4]. Since the movement of the mobile robot occurs in different directions, it is required to consider the longitudinal movement and rotation. Thus, in the structure of the friction model, it is required to provide the relationship between the sliding and twisting speeds [5]. An important development in the description of this relationship was made in [6]. Its author

managed to solve the equations for the principal moment and the vector of friction forces where a rectangle was considered as a contact area. Such analytical expressions enable to most accurately describe the motion of bodies on rough surfaces, but they are cumbersome and complex since they contain integral expressions. Hence, the authors of [7] constructed linear fractional Pade approximations, which made it possible to find solutions for the resulting dependences.

Pade approximation can be used to explain the effects of combined dry friction for linear and angular velocities. On the basis of Pade approximations, it became possible to create new models of friction [8, 9], which later began to be classified for better interpretation [10]. The classification occurs depending on the number of parameters. Thus, in [11], the authors introduced the notions of dimension and order of the dry friction model depending on the order of the used Pade approximations.

The model of sliding and twisting friction, which is proposed in the paper [12], provides considering the dynamic connection of the components that determine the force interaction of a rectangular body and a horizontal surface [13].

Problem Statement. We consider a solid body of mass m_0, which is a rectangular body with uniform faces of length a, width b and height $2h$. A fixed coordinate system $Oxyz$, associated with the body (Fig. 1) is introduced. Point O is located on the horizontal plane. The system. $O_1x_1y_1z_1$ starts at a point O_1, that corresponds to the geometric center of the body. The axis O_1z_1 is parallel to the axis Oz. The axis O_1x_1 is parallel to the long edge of the body. We introduce the unit vectors e_x, e_y of the axes O_1y_1 and O_1x_1, respectively.

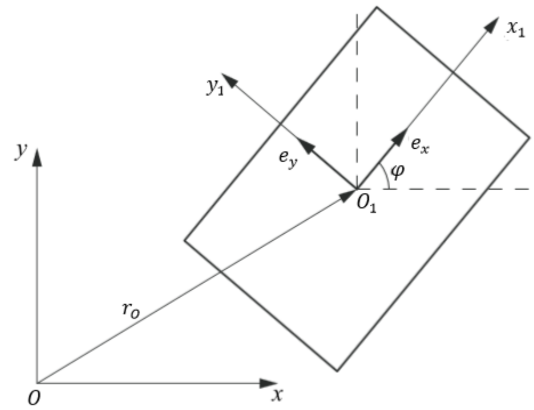

Fig. 1. Coordinate systems

Consider the continuous motion of the body on the surface (Fig. 2), which consists of translational movement and rotation about the axis O_1z_1. Three coordinates determine the position of the body. The coordinates x_0, y_0 and h set the origin of the coordinate system $O_1x_1y_1z_1$ in the coordinates $Oxyz$. The rotation of the body relative to its initial position on the axis O_1x_1 is specified by the angle φ. This paper considers the case when the center of mass of the body G and the center of mass of the system O_1 coincide (Fig. 2) [14].

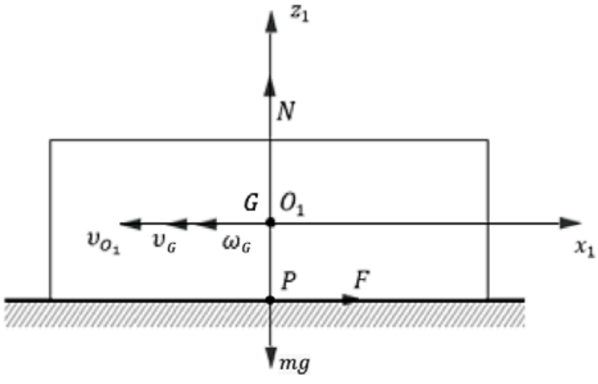

Fig. 2. Movement of the system body

Materials and Methods. The contact area is a rectangle with sides a and b, in which the normal voltage depends on the distance from the point P to the faces of the rectangle (Fig. 3).

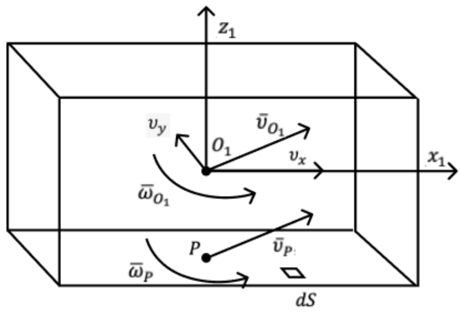

Fig. 3. Velocities of the points O_1 and P

Consider an infinitesimal area dS at an arbitrary point M on the contact surface. We introduce the angle φ between the relative sliding velocity and the axis O_1x_1. Let us draw the radius vector $\bar{r}_{MP}$ from the point P to the point M. The velocity vector of the point M is denoted υ_M, and to find it, we use the Euler formula describing the velocity distribution in a perfectly rigid body:

$$\upsilon_M = \upsilon_P + \omega \cdot \bar{r}_{MP}.$$

The sliding speed at the point M is decomposed into two components along the axes O_1x_1 and O_1y_1:

$$\upsilon_{Mx} = \upsilon_x - y\omega;$$

$$\upsilon_{My} = \upsilon_y + x\omega.$$

Using Coulomb's law, we find a small increment of the friction force directed against the relative velocity at the point M [15]:

$$d\bar{F} = -f\sigma(x,y)\frac{\upsilon_M}{|\upsilon_M|}dS,$$

where f — coefficient of friction; $\sigma(x,y)$ — contact stress distribution function depending on the x and y coordinates; $dS = dxdy$ — small area increment [15].

We rewrite the differential of the friction force and the moment of this force in projections on the axes under consideration:

$$dF_x = -f\sigma(x,y)\frac{\upsilon_{Mx}}{|\upsilon_M|}dxdy;$$

$$dF_y = -f\sigma(x,y)\frac{\upsilon_{My}}{|\upsilon_M|}dxdy;$$

$$dM_z = \begin{bmatrix} i & j & k \\ x & y & 0 \\ dF_x & dF_y & 0 \end{bmatrix} = xdF_y - ydF_x.$$

As a special case, we consider a uniform distribution of stress in the absence of internal masses in the body, then these stresses will be equal to: $\sigma = \frac{m_0 g}{ab}$, but then we will continue the record in general form: $\sigma(x,y)$.

Having integrated the expressions for the friction forces, we obtain:

$$F_x = -f\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}\sigma(x,y)\frac{\upsilon_x - y\omega}{|\upsilon_M|}dxdy; \quad (1)$$

$$F_y = -f\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}\sigma(x,y)\frac{\upsilon_y + x\omega}{|\upsilon_M|}dxdy. \quad (2)$$

Relative slip module $|\upsilon_M|$ is calculated from the formula:

$$|\upsilon_M| = \sqrt{\upsilon_{Mx}^2 + \upsilon_{My}^2} = \sqrt{\upsilon_x^2 + \upsilon_y^2 + \omega^2(x^2+y^2) + 2\omega(\upsilon_y x - \upsilon_x y)}. \quad (3)$$

Imagine the relative positions of the vectors of variable sliding speed υ and the components of the friction force: $F_{\parallel}$ — the component opposite to the sliding speed υ; $F_{\perp}$ — the component perpendicular to the instantaneous slip velocity. At the same time, imagine the coordination of this system with respect to the axes O_1x_1 and O_1y_1 (Fig. 4).

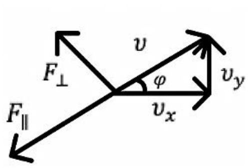

Fig. 4. Components of the friction force and velocity

We will make a transition from the projections of the sliding speed:

$$\begin{cases} \upsilon_x = \upsilon\cos\varphi, \\ \upsilon_y = \upsilon\sin\varphi, \end{cases} \tag{4}$$

to the speed module and the sliding angle:

$$\begin{cases} F_{\parallel} = F_x\cos\varphi + F_y\sin\varphi, \\ F_{\perp} = F_x(-\sin\varphi) + F_y\sin\varphi^{\cdot} \end{cases} \tag{5}$$

We will integrate the moment of the friction force on the contact area:

$$M_z = -f\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}\sigma(x,y)\frac{\upsilon(x\sin\varphi - y\cos\varphi)+\omega(x^2+y^2)}{\sqrt{\upsilon^2+\omega^2(x^2+y^2)+2\omega\upsilon(x\sin\varphi-y\cos\varphi)}}dxdy.$$

Let us substitute the expressions (1)–(3) into the system (5), and also rewrite the expression for the moment of force. As a result, we obtain a three-dimensional model of friction sliding and twisting:

$$F_{\parallel} = -f\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}\sigma(x,y)\cdot\frac{\upsilon(\cos^2\varphi+\sin^2\varphi)-\omega(y\cos\varphi+x\sin\varphi)}{\sqrt{\upsilon^2+\omega^2(x^2+y^2)+2\omega\upsilon(x\sin\varphi-y\cos\varphi)}}dxdy; \tag{6}$$

$$F_{\perp} = -f\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}\sigma(x,y)\cdot\frac{\omega(y\sin\varphi+x\cos\varphi)}{\sqrt{\upsilon^2+\omega^2(x^2+y^2)+2\omega\upsilon(x\sin\varphi-y\cos\varphi)}}dxdy; \tag{7}$$

$$M_z = -f\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}\sigma(x,y)\cdot\frac{\upsilon(x\sin\varphi-y\cos\varphi)+\omega(x^2+y^2)}{\sqrt{\upsilon^2+\omega^2(x^2+y^2)+2\omega\upsilon(x\sin\varphi-y\cos\varphi)}}dxdy. \tag{8}$$

In order not to solve cumbersome integrals, we use the replacement of the corresponding Pade expansions [16, 17]. Thus, based on the Pade theory [18], these expressions can be formulated as the ratio of two functions of several variables in the entire domain of definition, provided that the functions must have the same order [7]. To define these functions, it is required to determine the behavior of integral expressions (6)–(8) under the following conditions:

$$\frac{\partial F_{\parallel}}{\partial\upsilon}_{|\upsilon=0} = -\frac{f}{\omega}\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}\sigma(x,y)\cdot\frac{\sqrt{x^2+y^2}(x^2+y^2)+(y\cos\varphi+x\sin\varphi)(x\sin\varphi-y\cos\varphi)}{(x^2+y^2)^2}dxdy = -\frac{f}{\omega}I_0;$$

$$\frac{\partial M_z}{\partial\upsilon}_{|\upsilon=0} = -\frac{f}{\omega}\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}\sigma(x,y)\cdot\frac{(2y^2x\sin\varphi-2y^3\cos\varphi)}{(x^2+y^2)^2}dxdy = -\frac{f}{\omega}I_3;$$

$$M_{z|\omega\to\infty} = -f\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}\sigma(x,y)\cdot\frac{x^2-y^2}{\sqrt{x^2+y^2}}dxdy = -fI_6;$$

$$F_{\perp|\omega\to\infty} = -f\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}\sigma(x,y)\cdot\frac{y\sin\varphi+x\cos\varphi}{\sqrt{x^2+y^2}}dxdy = fI_9;$$

$$\frac{\partial F_{\parallel}}{\partial\upsilon}_{|\omega=0} = \frac{\partial F_{\parallel}}{\partial\omega}_{|\upsilon=0} = \frac{\partial F_{\parallel}}{\partial\omega}_{|\omega=0} = F_{\parallel|\omega\to\infty} = \frac{\partial F_{\perp}}{\partial\upsilon}_{|\omega=0} = \frac{\partial F_{\perp}}{\partial\omega}_{|\upsilon=0} =$$

$$= F_{\perp|\upsilon\to\infty} = \frac{\partial F_{\perp}}{\partial\omega}_{|\omega=0} = \frac{\partial M_z}{\partial\upsilon}_{|\omega=0} = \frac{\partial M_z}{\partial\omega}_{|\upsilon=0} = M_{z|\upsilon\to\infty} = 0.$$

Values of the expressions $\frac{\partial F_{\perp}}{\partial \upsilon}_{|\upsilon=0}$ and $\frac{\partial M_z}{\partial \omega}_{|\omega=0}$ are not involved in finding the subsequent Pade approximants, therefore, their writing is omitted due to their cumbersomeness. The identical equality to zero is realized under the condition that the voltage σ is symmetric about the center of the rectangular contact spot, i.e., the point P.

An accurate three-dimensional integral model [13] (6)–(8) provides a logical description of dry friction phenomena, but for solving problems of dynamics, such a model is difficult to accept due to the need to calculate impressive integrals [10]. To avoid this procedure, we use [6] to replace the exact integral system with the corresponding expressions using Pade approximations in the whole range of variables. Linear fractional Pade expansions give a three-dimensional model of first-order sliding and twisting friction [19]:

$$F_{\parallel} = F_0 \frac{\upsilon + b_1\omega}{\upsilon + d_1\omega}; \tag{9}$$

$$M_z = M_0 \frac{\omega + b_2\upsilon}{\omega + d_2\upsilon}; \tag{10}$$

$$F_{\perp} = F_0 \frac{\omega + b_3\upsilon}{\omega + d_3\upsilon}. \tag{11}$$

To determine the Pade coefficients, it is required to study the properties of this model at the boundary points by analogy with integral expressions. To do this, we differentiate the parameters $F_{\parallel}$, $F_{\perp}$, M_z and thus satisfy the corresponding integral expressions:

$$\begin{cases} F_{\parallel} = -fI_1 \dfrac{\upsilon}{\upsilon + \dfrac{I_0}{I_1}\omega} \\ M_z = -fI_6 \\ F_{\perp} = -fI_9 \dfrac{\omega}{\omega + \dfrac{I_3}{I_9}\upsilon} \end{cases}.$$

The system of equations of motion has the form:

$$\begin{gathered} J\frac{d\omega_{O_1}}{dt} = M_z; \\ (m_0+m_1)\frac{d\upsilon_x}{dt} = F_x + (m_0+m_1)\upsilon_y\omega_{O_1}; \\ (m_0+m_1)\frac{d\upsilon_y}{dt} = F_y - (m_0+m_1)\upsilon_x\omega_{O_1}. \end{gathered} \tag{12}$$

We express the time derivatives of the sliding speed and the angular speed using the formulas (3)–(5):

$$\frac{d\upsilon}{dt} = \frac{1}{2\sqrt{\upsilon_x^2+\upsilon_y^2}}\left(2\upsilon_x\frac{d\upsilon_x}{dt} + 2\upsilon_y\frac{d\upsilon_y}{dt}\right);$$

$$\frac{d\varphi}{dt} = \frac{1}{m_0+m_1}\left(-\frac{\upsilon\sin\varphi}{\upsilon^2}F_x + \frac{\upsilon\cos\varphi}{\upsilon^2}F_y\right).$$

We rewrite these equations using the formulas (3)–(5) for $\omega_{O_1} = \omega/a$ and add the first equation from the system (12):

$$\begin{gathered} J\frac{d\omega}{dt} = M_z a; \\ (m_0+m_1)\frac{d\upsilon}{dt} = F_{\parallel}; \\ (m_0+m_1)\upsilon\dot{\varphi} = F_{\perp}. \end{gathered} \tag{13}$$

Research Results. Next, we calculate the integral expressions of the parameters I_0, I_1, I_3, I_6, I_9 c using the Wolfram Mathematica software package for the following values:

$f = 1$; $a = 0.5$ m; $b = 0.2$ m; $m_0 = 1$ kg, $\sigma = {}^{m_0 g}/_{ab} = 87\,{}^{\text{kg}}/_{\text{s}^2\text{m}}$

and substitute in the system of equations (13). Based on numerical expressions, we build graphs of integral and normalized functions depending on the parameter $= {}^{\upsilon}/_{\omega}$. Fig. 5 shows graphs of the functions of the integral friction els (11)–(13), as well as models based on Pade approximations (14)–(16).

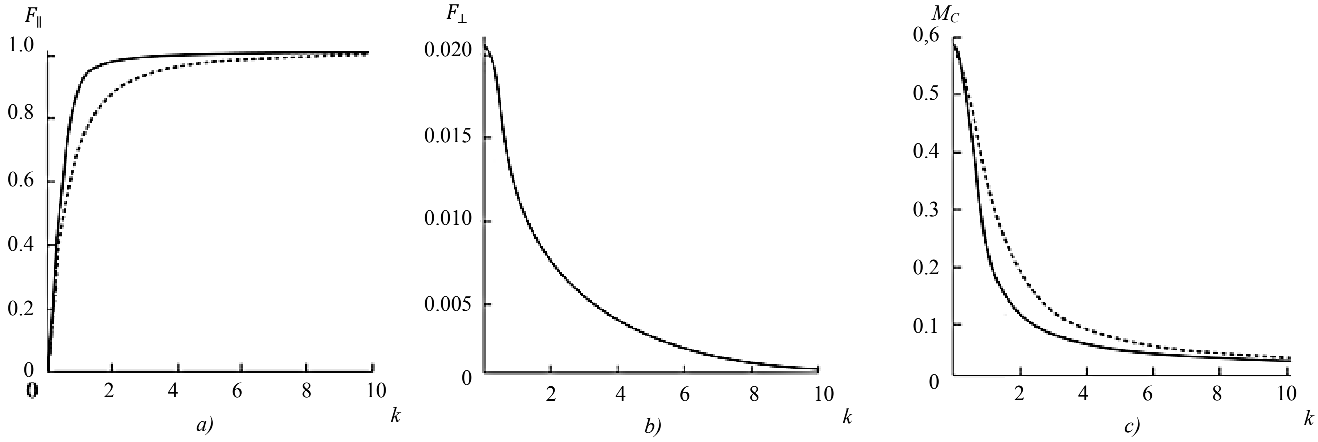

Fig. 5. Graphs of integral (solid lines) and normalized (dotted lines) functions of the tangent (*a*), normal (*b*) components of friction force and friction moment (*c*)

Based on the graphs of the functions (Fig. 5), we can talk about good matching of the considered models. Next, we obtain graphs of the dependences of the characteristic parameters on time (Fig. 6).

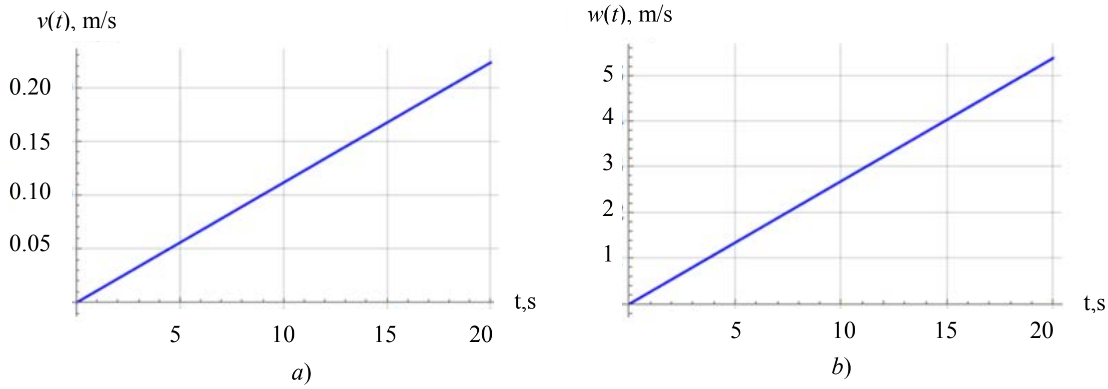

Fig. 6. Dependences of sliding velocity v (a) and angular velocity *w* (b) on time *t*

Discussion and Conclusions. The movement of the mobile robot, starting from the contact of its body and the reference plane, under the conditions of combined dynamics, when there is sliding and twisting, is described. Analytical integral expressions are obtained for the tangent and normal components of the friction force [19] and the moment of friction applied to a rectangular contact area. The corresponding Pade approximations are determined for the obtained expressions. The integral and normalized models are compared through plotting the dependences of the friction forces and the moment of friction on the angular velocity and the slip velocity. The comparison results showed good matching of the integral model and the model based on the Pade decompositions. The graphs correspond to the logical behavior when a rectangular body moves, since the sliding speed and angular velocity increase according to the specified parameters. Consequently, the combined friction model implemented using Pade approximations can be applied to solve problems related to mobile robots with a rectangular base.

References

1. Salimov MS, Ramzin NS. Dvizhenie tela na vibriruyushchei poverkhnosti v sluchae sukhogo treniya [Element movement on vibrating surface in case of dry friction]. Engineering and Automation Problems. 2019;4:100–104. (In Russ.)

2. Munitsyn LV. Vibrations of a Rigid Body with Cylindrical Surface on a Vibrating Foundation. Mech. Solids. 2017;52(6):675–685.

3. Voldrich J. Modelling of the three-dimensional friction contact of vibrating elastic bodies with rough surfaces. Appl. Comput. Mech. 2009;3(1):241–252.

4. Chowdhury MA, et al. Sliding friction of steel combinations. Open Mech. Eng. J. 2014;8(1):364–369.

5. Martinovs A, Gonca V. Descriptive model of sliding friction processes. Vide. Tehnologija. Resursi. = Environment. Technology. Resources. 2009;2:227–233.

6. Zhuravlev VF. O modeli sukhogo treniya v zadache kacheniya tverdykh tel [The model of dry fiction in the problem of the rolling of rigid bodies]. Journal of Applied Mathematics and Mechanics. 1998;62(5):762–767. (In Russ.)

7. Andronov VV, Zhuravlev VF. Sukhoe trenie v zadachakh mekhaniki [Dry friction in mechanical problems] Moscow. Izhevsk: R&C Dynamics; 2010. 184 p. (In Russ.)

8. Acary V, Brémond M, Huber O. On solving contact problems with coulomb friction: Formulations and numerical comparisons. In: Transactions of the European Network for Nonsmooth Dynamics on Advanced Topics in Nonsmooth Dynamics. Springer International Publishing; 2018. P. 375–457.

9. Haslinger J, Kučera R, Sassi T. A domain decomposition algorithm for contact problems with Coulomb's friction. Lect. Notes Comput. Sci. Eng. 2014;98:889–897.

10. Kireenkov AA. Svyazannaya model' treniya skol'zheniya i vercheniya [Related model of sliding and twisting friction]. Doklady Akademii Nauk. 2011;441(6):750–755. (In Russ.)

11. Zhuravlev VF, Kireenkov AA. O razlozheniyakh Pade v zadache o dvumernom kulonovom trenii [Pade expansions in the two-dimensional model of Coulomb friction]. Mechanics of Solids. 2005;2:3–13. (In Russ.)

12. Santos AP, et al. Granular packings with sliding, rolling, and twisting friction. Phys. Rev. E. 2020;102(3):032903.

13. Kireenkov AA. Trekhmernye modeli treniya [Three-dimensional friction models]. Vestnik of Lobachevsky University of Nizhni Novgorod. 2011;4(2):174–176. (In Russ.)

14. Sakharov AV. Povorot tela bez vneshnikh dvizhitelei pri pomoshchi rotora A. V. [Rotation of a body without external propellers using rotor]. Trudy MFTI. 2014;6(2):80–91. (In Russ.)

15. Zhuravlev VF. Zakonomernosti treniya pri kombinatsii skol'zheniya i vercheniya [Friction laws in case of combination of slip and spin]. Mechanics of Solids. 2003;4:81–89. (In Russ.)

16. Kireenkov AA. Zakon Kulona v obobshchennoi differentsial'noi forme v zadachakh dinamiki tverdykh tel s kombinirovannoi kinematikoi [Coulomb law in generalized differential form in problems of dynamics of rigid bodies with combined kinematics]. Mechanics of Solids. 2010;2:15–26. (In Russ.)

17. Gluzman S, Yukalov VI. Self-similarly corrected Pade approximants for nonlinear equations. Int. J. Mod. Phys. B. 2020;33(29):1–23.

18. Baker GA, Graves-Morris P. Pade approximations. In: Encyclopedia of Mathematics and its Applications, Series no. 59, 2nd ed. Cambridge University Press; 1996. 764 p.

19. Kireenkov AA. Svyazannaya model' treniya skol'zheniya i kacheniya v dinamike tel na sherokhovatoi ploskosti [Combined model of sliding and rolling friction in dynamics of bodies on a rough plane]. Mechanics of Solids. 2008;3:116–131. (In Russ.)

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Claimed contributorship

M. S. Salimov: basic concept formulation; research objectives and tasks; computational analysis; text preparation; formulation of conclusions. I. V. Merkuriev: academic advising; analysis of the research results; the text revision; correction of the conclusions.

All authors have read and approved the final manuscript.